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To determine the effect of temperature, the behavior of zinc, glass, ebonite, pitch and wax was studied. The results, together with the preceding may be summarized in the following formulae:*

$$\begin{aligned}
 S &= A_1 + A_2 T_2 + A_3 T_3, \\
 A_1 &= C_1 P e^{h_1 P}, & C_1 &= E_0 + E_1 e^{K_1 \theta}, \\
 A_2 &= C_2 P e^{h_2 P}, & C_2 &= E_2 e^{K_2 \theta}, \\
 A_3 &= C_3 P e^{h_3 P}, & C_3 &= E_3 \theta (T - \theta)^{-1}, \\
 T_2 &= 1 - e^{-\alpha \sqrt{t}}, & h &= b \theta, \\
 T_3 &= \left(t + \frac{A_0}{A_3} t_0 \right)^\rho - \left(\frac{A_0}{A_3} t_0 \right)^\rho, \quad \rho = \rho_0 + \frac{a}{1 + \left(\frac{\pi}{P \theta} \right)^m};
 \end{aligned}$$

S = displacement (twist), P = applied torque,
 t = time, t_0 = duration of previous stress,
 θ = temperature, T = melting point,
 $h, k, \alpha, E, P_0, a, b, m, \pi$, constants.

* Instead of this series coupling, the following may be substituted: The unit consists of four elements: (1), (2) and (3) are in viscous contact with (4). (1) and (2) are in elastic coupling; and finally (3) of this unit is connected with (1) of the next following unit by an elastic coupling. The resulting formulae, however, are not essentially different from those here given.

A NEW EQUATION OF CONTINUITY

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Communicated by A. A. Noyes, February 28, 1917

This paper presents a pressure, volume, temperature relation which has been carefully compared with the greater part of the available experimental data during the past ten years. The equation, valid when only one type of molecule is present, is

$$p = \frac{R}{v - \delta} T - \frac{a}{(v - l)^2} \quad (1)$$

where $\log \delta = \log \beta - \alpha/v$, and β, α, a , and b are constants; R is the universal gas constant.

The equation is based on considerations resulting from the inferences regarding atomic structure, obtained since the discovery of the negative electron. The model atom consists of a positive central portion about

which electrons are revolving. The negative electrons are assumed to possess a charge in the aggregate equal and opposite to the positive central charge. This model atom is essentially the atom discussed by Nicholson and Bohr.

Imagine an assemblage of such model atoms contained within an envelope and subject to their mutual attractive forces. The revolving electrons of a given atom generate a magnetic field and for convenience, therefore, the revolving electrons may be conceived as circles of current about the positive central portion of the atom. The positive central portion of one such atom repels the positive charge of another atom in a manner varying inversely as the square of the distance, and similarly the negative charges repel as r^{-2} . The negative charges attract the positive also according to r^{-2} ; but since, if all of the atoms were suddenly fixed in position their average distances apart would be equal, it may be assumed that the attractions and repulsions of the positive and negative charges mutually cancel. The potential due to the fields of the revolving electrons however do not cancel and therefore on the whole the resultant attraction in the assemblage of particles varies approximately as r^{-4} .

The attempt to calculate the complete expression for the force between two such model atoms is difficult and will require a more detailed knowledge of the atom. Sufficient is known, however, to make it certain that the attraction increases slightly more rapidly than r^{-4} .

An envelope of volume v and containing n particles would provide a space for the habitation of each particle on the average equal to v/n , so that the distance apart of the molecules, if a is the radius of the spherical envelope, would be equal to $\sigma = a(4\pi/3n)^{1/3}$. Therefore the potential energy would be proportional to $\frac{1}{2} \Sigma \Sigma \sigma^{-3}$ or equal to h/v where h is a suitable constant. The attraction then directed toward the center, would be per unit of surface identical in form to van der Waals' cohesive pressure a/v^2 . Now if the attractive force really increases slightly more rapidly than r^{-4} as appears to be really the case, it is easily perceived that a/v^2 would only be valid at very large volumes. A simple device to correct the expression is to assume that Φ , the cohesive pressure, may be represented by $a/(v-l)^2$, where l is a small constant which will of course make Φ increase more rapidly with diminution of volume than a/v^2 in the ratio of $v^2/(v-l)^2$.

The truth of the cohesive pressure expression $\Phi = a/(v-l)^2$ can be determined only by experiment since a rigorously complete solution for the attraction of an assemblage of the model atoms appears to be impracticable. If then the volume function for Φ given, is in agreement

with experiment and the sign of l comes out positive and not negative, as there is good reason to suppose it should, increased faith in the validity of the fundamental thought will have been established.

The term $(v-b)$ of van der Waals has given rise to much discussion. It now appears that a rigorous application of van der Waals' mode of thought really leads to a different expression for the 'volume correction.' It is, however, $(v-b)$ which seems to be the correct form as judged by comparison with experiment. With potential forces existing between the particles, it is difficult to see how collisions occur in the sense of the classical kinetic theory, at any rate within the mass of the gas. Indeed for an assemblage of particles such as here imagined a collision, as conceived by the older theory would not occur in general, except possibly at the boundary. It would, however, still be necessary to take account of the finite size of the atoms or rather more likely a certain space the center of which was occupied by an atom. In the case of a monatomic gas this volume would be fixed and the b of van der Waals therefore a constant. The equation of state for a monatomic gas at those volumes and temperatures where association is excluded would accordingly be

$$p = \frac{R}{v - \beta} T - \frac{a}{(v - l)^2} \quad (2)$$

where β , a , l are constants and R is the absolute gas constant defined by $R = pv/T$, the equation of a perfect gas.

When two of the model atoms are united to form a diatomic molecule an effect comes into play which is absent so long as monatomic particles are alone considered. A diatomic molecule is assumed to be held together by forces due to the positive and negative charges and the magnetic effect of the field due to the rotating electrons. The distance apart of these diatomic molecules in general will be large compared to the distance separating the two atoms forming the di-atom; but in an assemblage of these di-atoms, the field due to the assemblage must react to vary the distance separating the two atoms of a diatomic molecule. This would be inferred from the fact that for a magnetic shell in a magnetic field the reaction is always such as to bring about a decrease in the mutual potential energy. This decrease in the molecular case under consideration would result in a shortening of the distance between the two atoms of the di-atom or a decrease in its apparent volume as the strength of the field due to the surrounding di-atoms increased. It appears at present an impossible task to calculate exactly the apparent volume corresponding to van der Waals' b from such a physical picture.

The original mode of guessing the form of function is accordingly given.

The value sought, denoted by δ , would be a function of the volume alone and such a function that at infinite volumes it becomes constant, since when large distances separate the di-atoms the effect of the field due to the surrounding molecules would be negligible. The value of δ for $v = \infty$ let it be assumed then is β , a constant. The rate of change of δ for changes in volume would presumably be a function of the field due to the assemblage of particles and the value of δ , since the effect is due to the mutual action of the forces holding the di-atoms together and the external field due to the diatoms conjointly. This may be expressed by writing $d\delta/dv = f(\delta F)$, where F is the force. Now the simplest assumption under the circumstances is to assume that $d\delta/dv = k\delta F$ where k is a constant. The value of F taken per unit of area is approximately a/v^2 and the primitive of the equation becomes then, $\log \delta = \log \beta - a/v$ where $ka = \alpha$ another constant, and β is of course the limiting value which δ assumes when v is infinite. The equation for a diatomic assemblage of particles becomes then equation (1).

The equation (1) arrived at by such crude methods has been found to accord with the facts in a surprisingly exact manner, but in the present paper I will exhibit the evidence which has to do with the gist of the whole argument underlying the deduction of equations one and two. A later paper will present a discussion of the equation from a more general point of view including a comparison with all the available experiment data for various substances. It accordingly suffices for the present purpose to point out that the whole thought is tested uniquely by comparing a diatomic system with a monatomic system. The value of δ for the monatomic substance reduces to the constant β independent of the volume while δ becomes a volume function when the system consists of diatomic particles alone.

Recently sufficient data have appeared for argon which enables the test to be applied, while the diatomic system nitrogen will serve to test the functional form of δ . The triatomic molecule carbon dioxide appears to follow the same functional form for both gas and liquid phases. It should be emphasized that only those regions are available as legitimate for comparison purposes in which one order of molecule is present. That is to say, there must be no doubling of the argon atoms to form diatoms or association of the nitrogen molecules within the region of experimental data where comparisons are made. Such regions are fortunately a part of the experimental data contained in the literature.

It is noted that when the volume is held constant equations one and two state that the pressure will increase linearly with the temperature.

This fact was observed and discussed by Ramsay and Young for ether and carbon dioxide. Amagat thought, from an examination of his own data, that the 'law' was nearly exact in the case of many substances, certainly not true, however, in the case of the alcohols and water. Unfortunately it is necessary to have very accurate pressure measurements, to test the rule and moreover the measurements have always been made along isotherms instead of along isobars. This makes it necessary to obtain the constant volume data by graphical means with the liability of introducing a trend into the experimental data. It is comparatively easy to measure temperatures and pressures, but volume measurements under pressure are extremely difficult. Every substance which may be used to serve as a container for the experimental substance expands with temperature and pressure increase and the expansion may be moreover a complicated function of the time.

There can scarcely be any doubt that the high pressure gauge used by Amagat for the measurements above 1000 atm. was more in error than his low pressure gauge. The values of $p v$ for nitrogen were plotted accordingly for the several temperatures from 0° to 200° , with v as one coordinate in the region up to 1000 atm. Values of $p v$ were read off at the different temperatures for various values of the volume thus yielding material for calculating the constant volume, pressures, and temperatures. These values were laid off on coordinate paper and the best straight line passed through the data. The equations of the several straight lines was corrected subsequently by calculating the deviations and constructing an additional graph by means of which a deviation equation could be evaluated and also any trend in the deviations detected.

The equations obtained were all of the form, $p = [\Psi T - \Phi]_v$, and by comparison with equation (1) $\Psi = R/(v - \delta)$ and $\Phi = a/(v - l)^2$ whence $\delta = v - R/\Psi$. The δ values may be treated best by using $\log \delta$ and $1/v$ as the coordinates of the graph since it is proposed to test the functional form $\delta = \beta e^{-\alpha/v}$. The Φ values were tested by writing $v = \sqrt{a/\sqrt{\Phi} + l}$. It is seen that v should be plotted with $1/\sqrt{\Phi}$ as abscissae. The graph for nitrogen is given in figure one and for argon in figure two. It should be stated that the data given by Amagat above 1000 atm. give values of $\log \delta$ which join on to the values in figure two, but begin to differ slightly in trend as the pressure becomes greater. In fact a straight line may be passed through the 1-1000 atm. series and the 1000-3000 atm. series with the lines intersecting at about 1000 atm. The effect is due undoubtedly to the inaccuracy of the high pressure gauge.

The graphs show the functional form of δ as related to volume changes is very accurately satisfied, while argon shows no trend. In the graph

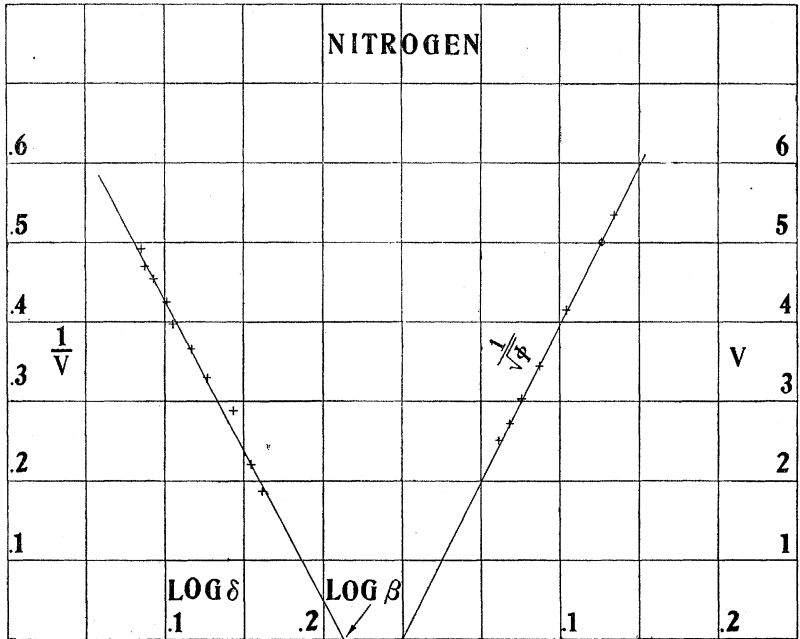


FIG. 1

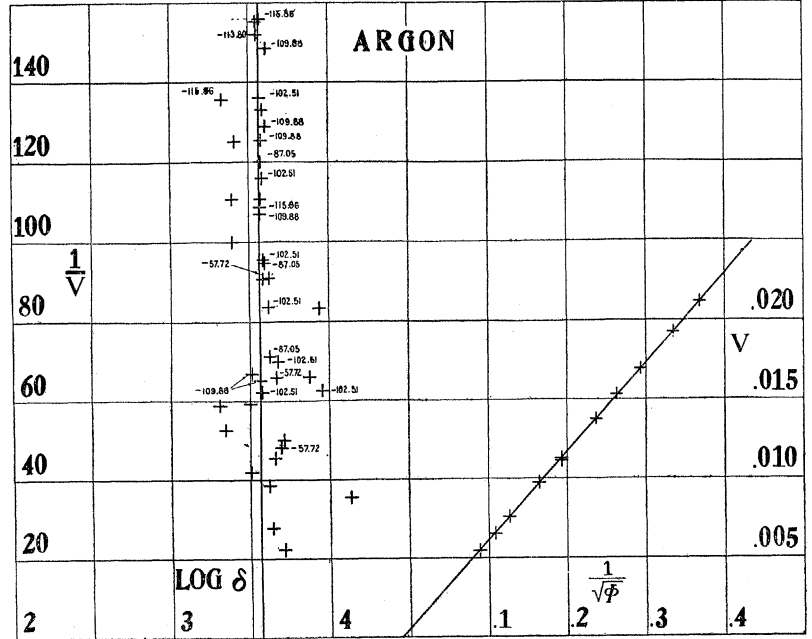


FIG. 2

TABLE 1
NITROGEN EQUATION OF STATE

v	$\rho, T=\phi$	$\rho, 16.03^\circ$	$\rho, 99.45^\circ$	$\rho, 199.5^\circ$
5.354	149.8	161.8	225.4	299.7K
	150.0	162.0	225.0	299 A
4.150	199.8	217.2	306.6	414.0
	200.0	217.0	307.0	414.0
3.460	250.4	273.1	390.3	531.3
	250.0	273.0	392.0	530.0
3.024	299.7	327.6	472.4	646.4
	300.0	328.0	474.0	644.0
2.729	347.7	381.1	552.9	759.5
	350.0	383.0	556.0	758.0
2.511	396.0	434.4	632.8	870.8
	400.0	439.0	637.0	869.0

The volume of 1 gram as determined by experiment is 799.1. This volume substituted in the equation of state gives 1.0004 instead of 1.0000.

$$p = \frac{2.9139}{v - \delta} T - \frac{1587.2}{(v - 0.007)^2}$$

$$\log_{10} \delta = 0.22001 - \frac{0.2839}{v}$$

TABLE 2
COMPARISON OF CALCULATED PRESSURES WITH PRESSURES GIVEN BY CROMMELIN FOR ARGON

TEMPERATURE	p_{cal}	p_{obs}	v	TEMPERATURE	p_{cal}	p_{obs}	v	TEMPERATURE	p_{cal}	p_{obs}	v
+20.39	61.932	61.741	0.016876	-113.8	42.692	42.682	0.009374	-119.2	37.753	37.641	0.010327
0°	31.478	31.572	0.030958						38.219	37.923	0.0101150
0°	36.671	36.61	0.026467						43.304	43.006	0.008002
-57.72	34.994	35.127	0.020782		52.074	51.752	0.006548		46.189	46.082	0.0069582
	45.876	46.209	0.015397		52.566	52.188	0.006435		47.95	47.272	0.0063954
	61.987	62.079	0.011026								
-87.05	33.231	33.296	0.017912	-115.86	41.932	41.908	0.009257				
	40.948	41.094	0.014003								
	51.431	51.533	0.010568								
	61.822	61.830	0.008345		55.13	53.204	0.0055576				
-102.51	28.150	28.840	0.019037		55.83	57.493	0.0054536	-120.24	37.801	37.836	0.010083
	32.375	32.394	0.016070						41.566	41.668	0.0084389
	35.552	35.784	0.014294						44.346	44.510	0.0073363
	40.808	40.976	0.011916	-116.62	37.171	37.250	0.011042				
	45.000	45.088	0.010438		41.811	41.943	0.009075				
	51.323	51.398	0.0086296		46.164	46.496	0.0074801				
	56.996	56.882	0.007372		50.45	50.259	0.006261	-121.21	37.588	37.465	0.0099676
	63.262	62.239	0.0063285						41.848	41.932	0.0080740
-109.88	31.532	31.515	0.015351						45.266	45.282	0.0067138
	32.051	31.929	0.015031								
	49.402	49.515	0.007964								
	54.475	54.250	0.006742								

of the latter substance the logarithm of δ was plotted with the density in order to bring out more clearly the relation of the various δ values to the density although an inspection of the tabulated values show that δ is constant. It might be added that helium, while the data are less extensive, gives identically the same result as does argon.

In tables 1 and 2 will be found the comparisons of the pressures calculated for the volumes and temperatures as given by Amagat for nitrogen and Crommelin for argon. The nitrogen pressures calculated show about as good an agreement throughout with the observed pressures as could be expected.

THE CLASSIFICATION OF VASCULAR PLANTS

By Edward W. Berry

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A scientific knowledge of plants or other natural objects consists to a very considerable extent in a knowledge of their mutual (phylogenetic) relations, hence the necessity of a Taxonomy that will consistently express filiation, which is only another way of stating that it should be phylogenetic. Science does not consist merely of names, but it cannot very well progress without a terminology, and not until this terminology becomes an expression of evolution can it become consistent and itself scientific.

Botanists have rather effectually grappled with this problem in the case of the lower plants, but the classification of the so-called vascular plants remains largely as an inheritance from the study of the end products of their evolution, namely a study of the existing vascular plants, with but slight consideration of the recent progress of paleobotanical investigation.

The not so very long obsolete practice of considering the Angiosperms and the Gymnosperms as subclasses of the class Exogens was a no more pernicious mask of their true relations than the extant usage which considers vascular plants as separable into two great phyla—the Pteridophyta or Sporophyta and the Spermatophyta. With the subdivisions of these two groups the present situation is equally inexpressive of our present state of knowledge. To the paleobotanist the Angiosperms and the Gymnosperms are obviously not groups of the same order, the latter including several groups of comparable rank with the Angiosperms as a whole, and extending over a period of time expressed by the ratio of 21 for gymnosperms to 6 for angiosperms.